11 Time Series

Learning Objectives

In this chapter you will learn:

- To identify types of variation and to investigate them by drawing a time series graph
- About seasonal variation and how to identify situations in which it occurs
- To show, by calculating centred moving averages, the trend in a set of data, and how to describe it
- How to calculate mean seasonal variation
- How to calculate estimates and predictions using seasonal components
- To appreciate the limited accuracy of the predictions that you make

Introduction

Time is a particularly important variable because so many others depend on it. At what time of day and on which day of the week would you advise a company to advertise its product on the radio? How would you expect the sales of umbrellas to vary from one **season** to the next?

Values of many variables depend on the time of year, or on some other time period. There may be a pattern in the way values vary over time, and if that pattern has some regularity to it then estimates and predictions can be made.

A time series is made up of measurements or readings of a variable taken at regular intervals of time.

The time intervals may be hourly, daily, weekly, monthly, seasonally or quarterly, termly, yearly, and so on.

The data is shown in a *time series graph*, which shows how the value of a variable has changed over a period of time. Time, as always, is the independent variable, so the time intervals are shown horizontally – usually along the longest side of the graph paper – and measurements or readings of the variable are shown vertically. Points are plotted and joined consecutively by ruled lines.

11.1 Variation and Trend

Variation refers to the changes in values from one time period to another, which are emphasised by the ruled lines that join the points on a time series graph.

If a pattern in the variation can be seen, it is perhaps best investigated by looking at changes in consecutive values of the variable; do they go up, down or remain constant?

The **trend** is a description of whether the value of the variable is increasing, decreasing or remaining constant over the whole period of time for which the readings have been taken.

You should not expect patterns in the variation to be perfectly regular.

Examples

1 Readings of a variable, X, are shown for 16 consecutive time periods in Table 11.1.

Table 11.1: Readings of X

Time period	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th	16th
Value of X	25	31	37	22	19	24	29	35	21	17	23	27	33	19	15	22

Looking at changes in values (U = up, D = down), we find the following:

UUDDUUUDDUUUDDU

This seems to have no pattern until it is broken down: UUDDU / UUDDU / UUDDU.

The pattern in the variation repeats after every five time periods.

Now we can investigate the trend by looking at sequences of every 5th reading.

The sequence for the 1st, 6th, 11th and 16th readings is: 25, 24, 23, 22, . . .

The sequence for the 2nd, 7th and 12th readings is: 31, 29, 27, . . ., and so on.

We can see that the values of X are decreasing.

2 Each term for four years, a student recorded her end-of-term examination mark in English – see Table 11.2.

Table 11.2: English examination marks

Year	2013			2014			2015			2016		
Term	1	2	3	1	2	3	1	2	3	1	2	3
Mark (%)	30	60	55	40	70	65	50	80	75	60	90	85

The data are shown in the time series graph in Figure 11.1.

Several important observations can be made from the graph:

- Her marks in English improved over time – this is the ‘trend’ shown by the data.
- The pattern in the variation of marks repeats every three terms, i.e. yearly.
- Marks were lowest in term 1 and highest in term 2 every year.

The variation and the trend allow us, by extrapolation, to predict what may happen in the future.

In doing this, we are assuming that the trend and variation continue in the same way.

By studying her marks, and assuming that the trend continues, we can predict her marks for 2017.

If her marks are set out in line with the pattern of variation, as in Table 11.3, the trend is clear to see.

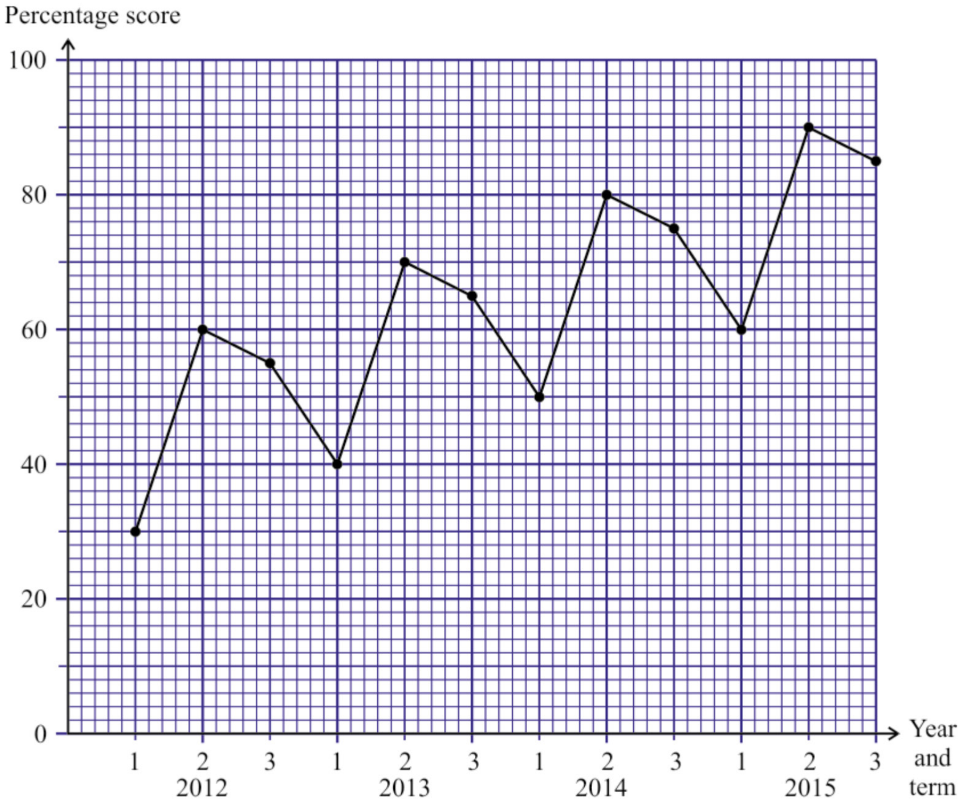

Figure 11.1: Time series graph of examination marks

Table 11.3: Alternative view of examination marks

	2013	2014	2015	2016	Prediction for 2017
Term 1	30	40	50	60	≈ 70
Term 2	60	70	80	90	≈ 100
Term 3	55	65	75	85	≈ 95

These predictions may not come true for any number of practical reasons, for example:

- Scoring 100% in an examination is highly unlikely.
- The student could fall sick at some point in time and perform badly.
- An examination could be cancelled.

Exercise 11A

1 The values of a variable *V* were recorded for ten time periods – see Table 11.4.

Table 11.4: Recorded values of V

Time period	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
V	2	5	3	6	4	7	5	8	6	9

i Draw a time series graph for the data, using 1 cm between time periods and 1 cm for 1 unit of *V*.
ii Describe the trend.
iii After how many time periods does the pattern in the variation repeat?
iv Study your graph and the table, and predict the value of *V* in the 11th and 12th time periods.

2 The values of a variable *Y* were recorded for 16 time periods – see Table 11.5.

Table 11.5: Recorded values of Y

Time period	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th	16th
Y	28	30	20	18	10	23	27	19	16	8	20	25	17	15	5	18

i Draw a time series graph for these data, using 1 cm between time periods and 2 cm for 5 units of *Y*.
ii Describe the trend.
iii After how many time periods does the pattern in the variation repeat?
iv Estimate the value of *Y* in the 17th and 18th time periods.

3 The consumption of electrical energy, in kilowatt-hours (kWh), at a bakery for each of the four seasons in a year for a period of $2\frac{3}{4}$ years is given in Table 11.6. (The four seasons in a year are often referred to as quarters.)

Table 11.6: Electricity consumption of bakery

Year	Season	Consumption (kWh)
2	I	27 500
0	II	22 500
1	III	15 000
3	IV	25 000
2	I	28 000
0	II	22 000
1	III	15 500
4	IV	24 000
2	I	27 000
0	II	23 000
1	III	14 500
5	IV	

i On graph paper with the longest side horizontal, draw and label an axis for time, using 2 cm between successive seasons, and an axis for energy consumed from 10 000 kWh to 30 000 kWh using 4 cm to represent 5000 kWh.
ii Use the data in the table to draw a time series graph.
iii Describe the trend in electricity consumption at the bakery.
iv During which season of each year does the bakery consume the greatest amount of electrical energy?
v By studying your graph and the table:
  a estimate the number of kilowatt-hours of electrical energy consumed in season IV of 2015,
  b suggest a practical reason why the estimate that you have made in (a) could be quite inaccurate.

4 Records from a hotel were used to calculate the percentage of its rooms that were occupied each night over ten six-month periods which, for each year, are April to September (summer) and October to May (winter). The data are presented in Table 11.7.

Table 11.7: Hotel occupancy

Year	2011		2012		2013		2014		2015	
Six-month period	Apr–Sep	Oct–May	Apr–Sep	Oct–May	Apr–Sep	Oct–May	Apr–Sep	Oct–May	Apr–Sep	Oct–May
Average occupation (%)	58	28	61	21	65	15	70	10	76	6

i Draw and label a horizontal axis for time with 2 cm between successive six-month periods and a vertical axis from 0% to 80%.

ii Plot the ten points from the table and draw a time series graph for the data.

iii Describe the variation shown in your graph.

The hotel manager predicts that the variation and trend will continue for at least five more years.

iv Give two reasons why the hotel manager's prediction cannot possibly come true.

11.2 Types of Variation

Variation in the values of a variable that depends on the time of year is known as **seasonal** variation – the pattern in the variation repeats after exactly one year. When variation is seasonal, the pattern could repeat after: 2 half-years, 4 quarters (seasons), 3 terms, 12 months, 52 weeks, 365 days, and so on.

Variation in the values of a variable that depends on any time period other than a year is known as cyclic variation. When variation is cyclic, the pattern could repeat after: 7 days, 24 hours, 6 months, 3 years, and so on.

Variation that does not fit into a particular pattern is said to be residual or random: the value of the variable at one point may be completely unexpected. Random variation is usually ignored unless it has a noticeable effect on future values.

The variation shown in a time series can be further described by its duration.

If the pattern in the variation repeats after N time periods, then the term *N-pointed* variation is used, where the word *pointed* refers to the time periods between readings.

For variables seen previously:

- Example 2: The student's examination marks in English have three-termly seasonal variation.
- Question 1: The variable V has two-weekly cyclic variation.
- Question 2: The variable Y has five-monthly cyclic variation.
- Question 3: Electricity consumption at the bakery has four-quarterly seasonal variation.
- Question 4: The percentage room occupation at the hotel has two-half-yearly seasonal variation.